

The Journal of Undergraduate Research

Volume 9 *Journal of Undergraduate Research*, Volume
9: 2011

Article 20

2011

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VIBRATIONAL ANALYSIS OF AN ELECTROMECHANICAL TOOTHBRUSH

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ABSTRACT

The effectiveness of an electromechanical toothbrush is based entirely on its ability to remove plaque through vibrations. This paper contests that the three-dimensional motion of more inexpensive models of electromechanical toothbrushes can be accurately modeled in one-dimension so as to predict the motion of future toothbrush designs using simply the material properties, motor specifications, and grip design. Vibrational modeling, mathematical modeling, and experimentation were used in the analysis to confirm the outcome of the study and the assumptions of the model. *Keywords:* vibration analysis, electromechanical toothbrush.

INTRODUCTION

Mechanical toothbrushes are becoming a common sight among household bathroom items. Closely following expensive models of any invention are more affordable versions which sometimes lack in quality. Colgate has developed many models of electromechanical toothbrushes which fit into the second category; however, using the science of vibrations and mathematical modeling it may be possible to design a toothbrush which is similar in cost, but opposite in quality.

To accomplish the design of an electromechanical toothbrush improved in quality, but without the increased price tag, a diagram must be constructed of the physical system. Once a diagram has been assembled, it may be decomposed and mathematical models can be constructed of the components. These mathematical models can be reconstructed allowing for a prediction of the vibrational motion of any toothbrush of similar design based solely on the materials on the toothbrush, the motor inside, and the design of the grip.

Standard engineering practice is to make certain accurate assumptions on a complex physical situation and to model that system according to those assumptions. Going a step further, a standard vibration principle is to take the developed model and simplify it until it can be solved through the most basic analysis. Once a model has been developed, it becomes necessary to confirm all assumptions through empirical data and observation.

METHODS

In an electromechanical toothbrush there occur three-dimensional vibrations due to the use of a single, unbalanced motor to force the vibrations. The removal of plaque, on the other hand,

is dependent only on one-dimensional motion parallel with the surface of a tooth. As such, this analysis assumed that the three-dimensional motion of a mechanical toothbrush could be accurately modeled as one-dimensional motion.

Figure 1 shows the breakdown of the physical toothbrush into its correlating diagram. To arrange the diagram in such a way that basic vibration mathematical models can be applied, the neck and head must be modeled together followed by the user's hand and then the impact of the user's hand and handle on the upper portions of the toothbrush.

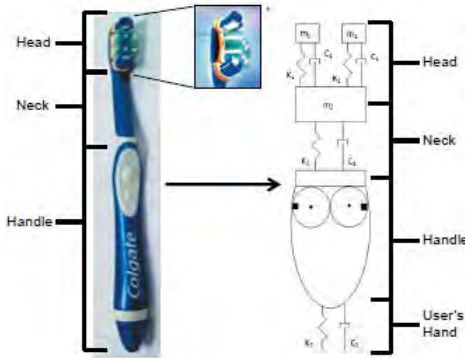


Figure 1. Diagram of Physical System [1]

Neck and Head

Beginning with the head and neck, the model is simplified through the process in Figure 2. The result of the supporting mathematical reduction of the head and neck are as follows:

$$m_{eq} = 2m_1 + m_2 \quad (1)$$

$$k_{eq} = \left(\frac{1}{2k_1} + \frac{1}{k_2} \right)^{-1} = \frac{2k_1 k_2}{2k_1 + k_2} \quad (2)$$

$$c_{eq} = \left(\frac{1}{2c_1} + \frac{1}{c_2} \right)^{-1} = \frac{2c_1 c_2}{2c_1 + c_2} \quad (3)$$

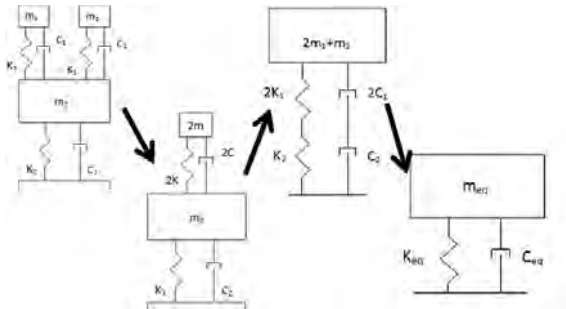


Figure 2. Head and Neck Diagram Reduction

In equation (1), m_1 and m_2 are mass of the head and the neck respectively; m_{eq} is equivalent mass of the head and the neck. In equation (2), k_1 and k_2 are the spring constants of the head and neck respectively while k_{eq} is the equivalent spring constant of the combined head and neck spring constants. Equation (3) is similar to the previous two equations where c_1 and c_2 are the damping coefficients of the head and neck respectively and c_{eq} is the equivalent damping coefficient of the head and neck.

Throughout the analysis in the paper, the use of terminology in the models will remain the same. The letter m represents a mass of some mechanical component, in this case, two small masses representing the multiple components on the toothbrush head seen in Figure 1 and the larger mass, m_{eq} , representing entirety of the mass from the hand grip to the head of the toothbrush. The letter k represents the vibrational property known as a spring constant. Springs are known for having a specific spring constant, but all materials have a spring constant value associated with them. The letter c represents the vibrational property known as the damping coefficient. This, like the spring constant, is both present in dampers as well as being a material property.

User's Hand and Handle

The vibration of the toothbrush is caused by an unbalanced motor near the grip of the toothbrush. This motor cannot, however, cause vibrations in free space. For vibrations to occur, the motor must be anchored to some fixed point. As the components of a toothbrush cannot be fixed to one another, the hand must act as the anchor. It is uncommon to consider the human body as a material, but for this analysis such a case must be examined. The spring constant and damping coefficient of the hand, k_3 and c_3 respectively, can be seen in Figure 3. The mathematical equations representing the unbalanced motor are seen below [4]:

$$M\ddot{y} + c_3\dot{y} + k_3y = me\omega^2 \sin(\omega t) \quad (4)$$

Resulting in

$$Y = \frac{me\omega^2}{[(k_3 - M\omega^2)^2 + (c_3\omega)^2]^{\frac{1}{2}}} \quad (5)$$

$$y(t) = Y \sin(\omega t - \phi) \quad (6)$$

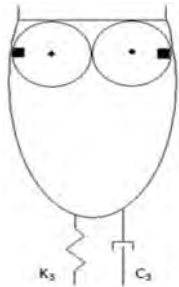


Figure 3. User's Hand and Handle

In equation (4), M represents the mass of the toothbrush while m represents the mass of the unbalanced mass. Eccentricity, or e , refers to the distance which the unbalanced mass's center of gravity is rotating from the shaft of the motor. The \ddot{y} , \dot{y} , and y terms refer to the acceleration, velocity, and displacement of the motor in a single plane of motion respectively. Finally, t represents time and the Greek letter ω refers to the angular velocity of the motor and toothbrush.

In equations (5) and (6), Y represents the amplitude of the vibration caused by the rotation of the unbalanced motor, ϕ the phase shift of the vibration (negligible for these considerations), and $y(t)$, the resulting function for the unbalanced motor vibration.

Final Equation

With the unbalanced motor interacting with the user's hand and the handle of the brush, an equation exists to model the motion of the lower portion of the brush. This harmonic motion in the base of the toothbrush must now be applied to the upper portion of the toothbrush as though the base is a simple plane experiencing a harmonic vibration. Figure 4 illustrates the harmonic motion and the resulting force in the upper portion of the brush.

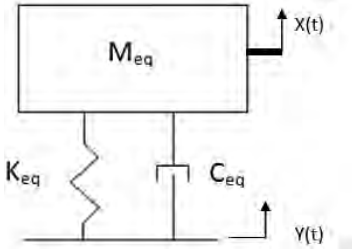


Figure 4. Base Vibration Forcing Upper Toothbrush

The following are equations describing this base vibration [4]:

$$m_{eq}\ddot{x} + c_{eq}(\dot{x} - \dot{y}) + k_{eq}(x - y) = 0 \quad (7)$$

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = k_{eq}y + c_{eq}\dot{y} = k_{eq}Y\sin(\omega t) + c_{eq}\omega Y\cos(\omega t) \quad (8)$$

$$k_{eq}Y\sin(\omega t) + c_{eq}\omega Y\cos(\omega t) = A\sin(\omega t - \alpha) \quad (9)$$

$$A = Y\sqrt{k_{eq}^2 + (c_{eq}\omega)^2} \quad (10)$$

$$\alpha = \tan^{-1}\left[-\frac{c_{eq}\omega}{k_{eq}}\right] \quad (11)$$

$$x(t) = A\sin(\omega t - \alpha) \quad (12)$$

In equations (7) through (12), \ddot{x} is the acceleration of the upper portion of the toothbrush, \dot{x} is the velocity of the same, and x is the displacement also of the upper portion

of the toothbrush. Also included is \dot{y} the velocity of the lower portion of the toothbrush, and y the displacement of the same.

Combining equations (1), (2), (3), (5), (6), (10), (11), and (12) yield the final solution, $x(t)$, which predicts the motion of a toothbrush from the masses, spring constants, and damping coefficients of the different components of an electromechanical toothbrush.

$$x(t) = \left[\left(\frac{me\omega^2}{[(k_3 - M\omega^2)^2 + (c_3\omega)^2]^{\frac{1}{2}}} \right) \sqrt{k_{eq}^2 + (c_{eq}\omega)^2} \right] \cdot \sin \left[\omega t - \left(\tan^{-1} \left[-\frac{c_{eq}\omega}{k_{eq}} \right] \right) \right] \quad (13)$$

ANALYSIS AND EXPERIMENTATION

Experimentation was performed to validate the mathematical model after its development. First, it was necessary to establish the spring constant, k_3 , and the damping coefficient, c_3 , of the hand. The measurement of such properties was beyond the capabilities of available equipment so the research of three professors from Johns Hopkins University was relied upon for those values. Normal values were chosen to represent the largest portion of the population through design. The measurement of hand properties using the testing apparatus shown in Figure 5, produced values of 0.9 N/mm for the spring constant and 0.006N-s/mm for the damping coefficient [3].



Figure 5. Hand Vibrational Properties Measuring Apparatus [3]

With hand values determined, the toothbrush was weighed using a precision scale finding the mass, M , of the toothbrush to be 35.81g. Also, the toothbrush was destructed and the eccentric motor weight removed for measurement using the same scale. This value proved to be 1.2g, much smaller than that of the toothbrush.

Next, the angular velocity at which the end of the toothbrush was traveling needed to be determined. This was accomplished using an Olympus i-Speed 3 high speed camera to observe the toothbrush's linear displacement in frames per second. A video frame can be seen in Figure 6.



Figure 6. High Speed Video Clip

In order to calculate the spring constant, k_{eq} of the toothbrush, it was held in a test fixture. Specific loads were applied to the end of the brush while recording the displacement of the brush using a Jeweled dial indicator with 0.005 inch precision as seen in Figure 7.



Figure 7. Measuring Spring Constant of Head and Neck

Although it is not often associated with vibrational analysis, the Young's Modulus, E , of the head and neck of the toothbrush may be used as another method for calculating the spring constant, k_{eq} . Available through the Materials Evaluation and Testing Laboratory, METLAB, at South Dakota State University was an MTS machine. This machine, as seen in Figure 8, allows for material samples to be stretched under increasing loads to plot the stress versus strain curve of the material. The beginning, linear, portion of the stress/strain curve is directly correlated with the Young's Modulus of that material. Figure 9 shows a plot of a sample toothbrush's stress/strain curve which yielded a Young's Modulus value of 4.077N/mm^2 .

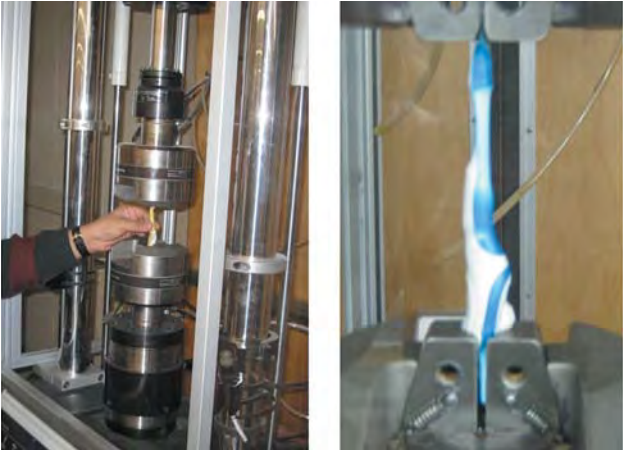


Figure 8. MTS Machine and Toothbrush Sample

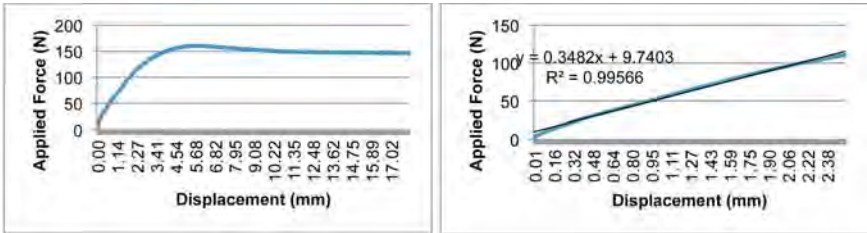


Figure 9. Head and Neck Stress/Strain Curve

With the final forcing function measured in the high speed video and all material values calculated except the damping constant of the head and neck, c_{eq} , the value for the damping constant could be solved from the equation. However, multiple data points needed to be produced in order to measure the error in the one-dimensional model and to verify the resulting value for the damping constant. To do this, the battery from the toothbrush was removed and a variable voltage power supply was supplemented in its place. Varying the voltage supplied to the motor would yield different angular velocities and resultant functions, both seen in the high speed video, but the value for the damping constant should remain the same. Figure 10 shows the result of a few sample tests in Engineering Equation Solver.

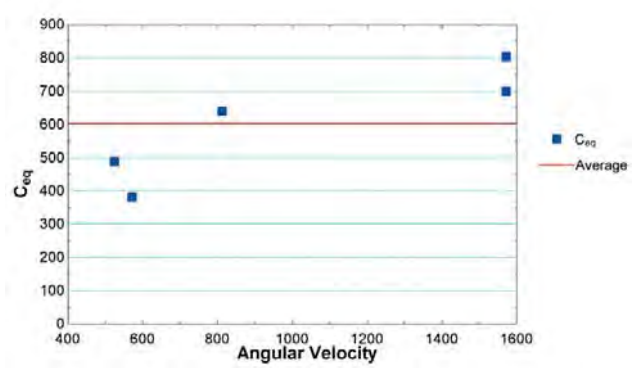


Figure 10. Damping Coefficient Test Results

RESULTS

The results of the values that were calculated from the equations of motion, high speed videos, and measured physical properties are seen in Table 1, but referring back to the data shown in Figure 10, there appears to be a large amount of error as the standard deviation is 169 N-s/m. However, three factors should be considered. First, damping coefficients are notorious for their difficulty to be measured. Geometry, material age, and sometimes even temperature change the damping coefficient. Second, all the errors occurring in the model show in the graph in Figure 10. Third, the neck of the toothbrush is a composite of both rubber and plastic, further complicating accurate material tests due to varying composite percentages.

Table 1. Property Test Results

Symbol	Property	Value
k_3	Hand spring constant	0.9 N/mm
c_3	Hand damping coefficient	0.006 N-/mm
M	Mass of toothbrush	0.03581 kg
m	Mass of eccentric weight	0.0012 kg
ω	Angular velocity	7758.62 rpm
e	Eccentricity of unbalanced motor	3 mm
k_{eq}	Head and neck spring constant	1.765 N/mm
c_{eq}	Head and neck damping coefficient	0.603 N-s/mm

Considering all of these factors as well as the fact that the rubber/ plastic composite is expected to yield results below the 1,000 N-s/m range, the results seem to confirm the

assumption that a one-dimensional model can accurately predict the motion of a three-dimensional situation [2].

Further confirming this data was an observation made during the filming of the high speed videos. As a force is applied to the bristles of the brush in a similar fashion to that experienced when brushing teeth, the obvious three-dimensional motion that was present in earlier videos disappears and all motion is converted to a linear motion that is the most conducive to plaque removal.

DISCUSSION

As a result of the extent to which experimentation was performed on multiple toothbrush samples, it can be concluded that the one-dimensional mathematical vibration model is a valid approximation of the actual three-dimensional situation. The experimental data confirmed this finding, allowing for reasonable error, and a visual observation of the resulting motion under force applied to the bristles yielded confirmation as well.

The American Dental Association suggests that for optimal plaque removal, a toothbrush should vibrate to a peak of 30,000 strokes per minute [5]. Using this information for toothbrush redesign would suggest that more money should be spent in specifying better motors while limiting the prices spend on other materials. Careful consideration of the mathematical model also suggests that grip design is a critical part of toothbrush design impacting not only the spring constant and damping coefficient of the hand, but also the resulting spring constant and damping coefficient of the head and neck.

LIMITATIONS

As previously discussed, extensive testing should be done when calculating the damping coefficient of the neck of a toothbrush. May issues can lead to complications in its measurement such as changes in geometry and component composition. Also, the creation of the vibrational model in this paper is only valid for design of toothbrushes of similar geometry which utilize an unbalanced motor as a means of creating mechanical vibrations through the structure of the brush. The model is invalid for all other electromechanical toothbrushes whose vibrations are powered by any other means.

ACKNOWLEDGEMENTS

Nathanael Rehn, Tara Jeatran, and Darin Waldner – Collaborators
Materials Evaluation and Testing Laboratory (METLAB) – Testing Equipment

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